MATH 320 Unit 4 Exam Name:

Instructions:

Please write your answers on separate paper. Please write clearly and legibly, using a large font and plenty of white space (I need room to put my comments). Staple all your pages together, with your problems in order, when you turn in your exam. Please do not write under the staple. Make clear what work goes with which problem. Put your name or initials on every page. To get credit, you must show adequate work to justify your answers. If unsure, show the work. No outside materials are permitted on this exam – no notes, papers, books, calculators, phones, smartwatches, or computers – only pens and pencils, and your coursepack. You may use any result in the coursepack (whether boxed or an exercise). However, you must cite it, and you may not use it to trivialize an exam question (e.g. to prove itself or a portion of itself or a special case of itself). Each problem is out of 10 points, 100 points maximum. You have 75 minutes.

1. Consider the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ given by $f((m, n)) = (n, m)$. Prove or disprove that f is a homomorphism.
2. Consider the function $g : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ given by $g((m, n)) = m + n$. Prove or disprove that g is a homomorphism.
3. Recall that $M_2(\mathbb{R}) = \left\{ \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right) : a, b, c, d \in \mathbb{R} \right\}$. Consider the function $h : M_2(\mathbb{R}) \to M_2(\mathbb{R})$ given by $h\left(\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)\right) = \left(\begin{smallmatrix} a & c \\ b & d \end{smallmatrix}\right)$. Prove or disprove that h is a homomorphism.
4. Prove that $\mathbb{Z}_9$ and $\mathbb{Z}_3 \times \mathbb{Z}_3$ are not isomorphic.
5. Prove the Ring Image Theorem.
6. Let R be a commutative ring with identity, and let $c \in R$. Prove that the principal ideal $(c) = \{rc : r \in R\}$ is indeed an ideal.
7. Let R be a commutative ring, and let I be an ideal. Prove that equivalence modulo I is transitive.
8. Consider the ring $\mathbb{Z}[x]$. Prove or disprove that $(2) \cap (x^2) = (2x^2)$.
9. Consider the ring $\mathbb{Z}[x]$, and its ideal $I = (2) + (x^2) = \{a + b : a \in (2), b \in (x^2)\}$. Prove or disprove that I is principal.
10. Consider the ring $\mathbb{Z}[x]$, and its ideal $I = (2) + (x^2) = \{a + b : a \in (2), b \in (x^2)\}$. Find R/I, i.e., find all of the cosets of I. How many are there?